Negative muon capture in deuterium

1974 J. Phys. A: Math. Nucl. Gen. 7 1454

(http://iopscience.iop.org/0301-0015/7/12/009)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 12, 1974. Printed in Great Britain. © 1974

Negative muon capture in deuterium

S Srinivasa Raghavan
Physics Department, College of Engineering, Guindy, Madras-600 025, India

Received 12 March 1974

Abstract. We calculate the total negative muon capture rate in deuterium using the closure approximation, and compare our results with those of Pascual *et al* who use the impulse approximation for computing the capture rate.

1. Introduction

Primakoff (1959) has worked out an extensive theory of muon capture in nuclei using the closure approximation and this was subsequently modified by Goulard *et al* (1964) to include relativistic corrections. In a recent publication, Pascual *et al* (1972) have computed the total negative muon capture rate in deuterium ($\Lambda^{(\mu)}({}^2_1\mathrm{H})$) in the impulse approximation, using the analytical wavefunctions of Gourdin *et al* (1965) for the ground state of the deuteron. The result of Pascual *et al* (1972) for the capture rate is in complete disagreement with that of Primakoff (1959). In order to investigate this discrepancy we compute the muon capture rate ($\Lambda^{(\mu)}({}^2_1\mathrm{H})$) in deuterium, in the closure approximation, using the wavefunctions of Gourdin *et al* (1965). We also use the 'realistic' deuteron wavefunctions obtained using Ueda and Green (1968) and Reid (1968) soft-core potentials for calculating $\Lambda^{(\mu)}({}^2_1\mathrm{H})$, in order to see the effect of these wavefunctions on the capture rate. The 'realistic' deuteron wavefunctions of Ueda and Green (1968) and Reid (1968) are found to yield good form factor data for the deuteron (Raghavan 1973) and reasonably good cross sections for the neutral pion photoproduction (Rao and Raghavan 1973) from the deuteron.

2. Calculation of $\Lambda^{(\mu)}({}^2_1\mathrm{H})$

In the closure approximation the muon capture rate in deuterium, when the relativistic corrections are included is given by (Goulard *et al* 1964)

$$\Lambda^{(\mu)}({}^2_1\mathrm{H}) = (Z_{\mathrm{eff}})^4(\langle\eta\rangle_{\mathrm{a}})^2(272\ \mathrm{s}^{-1})R\left[1-\left(\frac{(G_{\mathrm{V}}^{(\mu)})^2+(\Gamma_{\mathrm{A}}^{(\mu)})^2}{(G_{\mathrm{V}}^{(\mu)})^2+3(\Gamma_{\mathrm{A}}^{(\mu)})^2}\right)(\alpha_{\mathrm{a}}^{(+)}+\alpha_{\mathrm{a}}^{(-)})(1-x_{\mathrm{a}})\right] \tag{1}$$

in which

$$\alpha_{\mathrm{a}}^{(+)}+\alpha_{\mathrm{a}}^{(-)} = \iint j_0(\langle v\rangle_{\mathrm{a}}|\boldsymbol{r}_1-\boldsymbol{r}_2|)|\Phi_{\mathrm{a}}(\boldsymbol{r}_1,\boldsymbol{r}_2)|^2\delta(\boldsymbol{r}_1+\boldsymbol{r}_2)\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2. \tag{2}$$

In the above equations a denotes the ground state of the deuteron, and x_{a} is the relativistic correction.

In equation (1) the term $(\alpha_a^{(+)}+\alpha_a^{(-)})$ called the exclusion principle inhibition factor alone requires the explicit consideration of the ground state wavefunction of the deuteron; other terms can be calculated in terms of the known quantities (Goulard *et al* 1964). The derivation of equation (2) is based on the assumption that we can factorize the complete ground state wavefunction of the deuteron as

$$\psi_a = |a\rangle = \chi_a(\sigma_1^{(3)}, \sigma_2^{(3)}; \tau_1^{(3)}, \tau_2^{(3)})\Phi_a(\boldsymbol{r}_1, \boldsymbol{r}_2), \tag{3}$$

where χ_a is the spin isospin part of the wavefunction and Φ_a is the spatial part. If the spin isospin is normalized, equation (2) can be written as

$$\alpha_a^{(+)}+\alpha_a^{(-)} = \int j_0(\langle\nu\rangle_a|\boldsymbol{r}_1-\boldsymbol{r}_2|)|\psi_a|^2\delta(\boldsymbol{r}_1, \boldsymbol{r}_2)\,d\xi \tag{4}$$

in which the variable ξ denotes the assembly of position, spin and isospin coordinates. The validity of using equations (2) and (4) and the justification for satisfying the spherical symmetry of the wavefunction when the D-state admixture is included in the ground state wavefunction have been rigorously discussed by Srivastava *et al* (1972), for the 1s shell nuclei.

Our calculated values of $\Lambda^{(\mu)}({}^2_1\mathrm{H})$ (for $g_P^{(\mu)}/g_A^{(\mu)} = 8$, where $g_P^{(\mu)}$ and $g_A^{(\mu)}$ are the induced pseudoscalar and axial vector muon-dressed nucleon coupling constants) along with those of Pascual *et al* (1972) are shown in table 1. A comparison of the values obtained for $\Lambda^{(\mu)}({}^2_1\mathrm{H})$ with both the closure and the impulse approximations, for the case of the analytical wavefunctions of Gourdin *et al* (1965), shows that the value obtained with the impulse approximation is considerably less than that obtained with the closure approximation. The difference between the two values is 21 s^{-1}. It is really surprising to note such *a large difference* between the two methods. From table 1 we also see that the closure approximation is quite insensitive to the ground state wavefunctions used.

Table 1. Total μ^- capture rate in deuterium for $g_P^{(\mu)}/g_A^{(\mu)} = 8$.

Deuteron wavefunctions	Closure approximation (s^{-1})	Value of the integral $\alpha_a^{(+)}+\alpha_a^{(-)}$	Impulse approximation (s^{-1})
Gourdin *et al* (1965)	133·6	0·6444	112·2‡
Reid (1968)	134·6	0·6327	
Ueda and Green (1968)	136·9	0·6142	
Primakoff (1959)	135·0†	0·6400	

† Primakoff (1959) (non-relativistic treatment).
‡ Pascual *et al* (1972).

3. Conclusions

The values obtained by Pascual *et al* (1972) for $\Lambda^{(\mu)}({}^2_1\mathrm{H})$ clearly show that the effect of the D-state admixture is more important than the relativistic correction. However, in our calculations we find that the inclusion of both the D-state admixture and the relativistic corrections are insensitive to the capture rate. We are unable to compare our results with experiments, because no reliable experimental measurements for the muon capture rate in deuterium are available (Marshak *et al* 1969, Pascual *et al* 1972). Hence, until reliable experimental results are available it is very difficult to discriminate between the two methods (namely, closure and impulse approximations), which are used for computing the total muon capture rate for deuterium.

Acknowledgments

The author is very grateful to Professor B K Srivastava, Indian Institute of Technology, Kharagpur, for suggesting this problem. The help rendered by the computer staff of the College of Engineering, Guindy, is gratefully acknowledged.

References

Goulard B, Goulard G and Primakoff H 1964 *Phys. Rev.* **133** B186–90
Gourdin M, Le Bellac M, Renard F M and Tran Thanh Van J 1965 *Nuovo Cim.* **37** 524–42
Marshak R E, Riazuddin and Rayan C P 1969 *Theory of Weak Interactions in Particle Physics* (New York: Wiley International) p 297
Pascual P, Tarrach R, Vidal F 1972 *Nuovo Cim.* A **12** 241–8
Primakoff H 1959 *Rev. Mod. Phys.* **31** 802–22
Raghavan S S 1973 *Prog. Theor. Phys.* **49** 2135–7
Rao K S and Raghavan S S 1973 *Atomic Energy Commission Symp., Bangalore* (Bombay: Department of Atomic Energy)
Reid R V 1968 *Ann. Phys., NY* **50** 411–48
Srivastava B K, Mahanti D and Yadav H L 1972 *Nucl. Phys.* A **181** 217–24
Ueda T and Green A E S 1968 *Phys. Rev.* **174** 1304–12